FAR FIELD NOISE PREDICTION FROM NEAR FIELD SPECTRUM MEASUREMENTS

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ABSTRACT

A theory is presented for computing far field noise radiation from near field measurements. The relations are given for a spheroidal surface. It turns out that the same mathematical development in spheroidal functions can be used both for single frequency and for noise spectrum.

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ACKNOWLEDGEMENTS

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LIST OF SYMBOLS

$\Gamma_{12}(\vec{r}_1,\vec{r}_2,\tau)$	cross correlation function of the noise pressure at points $\vec{r}_1$ and $\vec{r}_2$
$\vec{r}_1$	position vector of a point in the far field
$\vec{r}_2$	position vector of a point in the near field
τ	correlation time
$p(\vec{r}_1,t)$	pressure at point $\vec{r}_1$ as a function of time
$p(\vec{r}_2,t-\tau)$	pressure at point $\vec{r}_2$ at time $t-\tau$
$\vec{r}_s$	position vector of a point on the surface over which the near field measurements are taken
r	distance from a point on the surface to the far field point
$\vec{n}$	outward normal to the surface
∇_1^2, ∇_2^2	operators which define the spatial part of the wave equation
$G_{12}(\vec{r}_1,\vec{r}_2,\nu)$	cross power spectrum of noise pressure at points $\vec{r}_1$ and $\vec{r}_2$
ν	frequency
$G_1(\vec{r}_1,\nu)$	power spectrum of noise pressure at $\vec{r}_1$
$G_{s2}(\vec{r}_s,\vec{r}_2,\nu)$	cross power spectrum between a point on the measurement surface $(\vec{r}_s)$ and a near field reference point $(\vec{r}_2)$
$\mathcal{G}_1$	Green's function for the Helmoltz equation which vanishes over the measurement surface
S	the surface over which measurements are taken
ξ, η, ϕ	spheroidal coordinates of a point in the far field $(\vec{r}_1)$
ξ_2, η_2, ϕ_2	spheroidal coordinates of a reference point in the near field $(\vec{r}_2)$
ξ_s, η_s, ϕ_s	spheroidal coordinates of a point on the measurement surface
δ_{om}	delta function of o and m ; 0 if $m \neq 0$, unity if $m = 0$
N_{mn}	normalization constant for spheroidal wave functions (see Flammer "Spheroidal Wave Functions p. 22, Formula 3.1.33)
$S_{mn}(c,\eta)$	spheroidal angle function (see Flammer, p. 16, Formula 3.1.3a)
$R_{mn}^{(1)}(c,\xi)$	spheroidal radial function (see Flammer, p. 32, Formula 4.1.16)

c	$\frac{1}{2} k d$
k	ω / c_0
ω	angular frequency
c_0	sound velocity in the medium
d	interfocal distance of the spheroid (see Flammer, p. 6-7)
m, n	numbers describing the order of the spheroidal function

I. Introduction

The determination of far field radiation patterns for sinusoidal signals calculated from pressure and phase measurements made near and around a vibrating transducer has been accomplished with great success by the group at the University of Texas.[1-4] The more general problem of computing the far field noise from a random noise source is one that is also of great practical significance. The theory of noise fields for systems that satisfy the wave equation has been developed independently by several investigators working in the fields of acoustics,[5,6] optics,[7,8] and electromagnetic theory.[9]

It is only recently that various investigators have been thinking about the problem of predicting the far field noise from measurements made near and around the source. Horton and Innis[10] have discussed the noise problem briefly in a supplement to their report. Marsh[11] has proposed a method using the cross correlation function and the Green's Function for general time variations,* and Ferris[12] has proposed using the relations derived by Parrent[8] to compute the far field from near field noise measurements. It will be shown in this report how the basic work of Parrent[8] can be employed to derive a method which is a variation of the Marsh procedure[11] and then how the work of Horton-Innis[2] can be used to complete the method for a finite closed surface.

II. Statement of the problem

Given a noise source and a method by which pressure time signals can be recorded near and around the source. The problem is to determine the power spectrum of the noise in the far field.

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1. C. W. Horton and G. S. Innis, J. Acoust. Soc. Am. <u>32</u>, 938(A) 1960.
2. C. W. Horton and G. S. Innis, J. Acoust Soc. Am., <u>33</u>, 877, 1961.
3. D. D. Baker and C. W. Horton, J. Acoust Soc. Am., <u>33</u>, 836(A) 1961.
4. D. D. Baker, J. Acoust. Soc. Am., <u>34</u>, 1737, 1962.
5. H. W. Marsh, "Correlation in Wave Fields," USL Quarterly Report, Oct.-Dec., 1949, U. S. Navy Underwater Sound Laboratory.
6. C. Eckart, J. Acoust. Soc. Am., <u>25</u>, 195, 1953.
7. E. Wolf, Proc. Roy. Soc. A, <u>230</u>, 246, 1955.
8. G. B. Parrent, Jr., J. Optical Soc. Am., <u>49</u>, 787, 1959.
9. P. Roman and E. Wolf, Nuovo Cimento, <u>17</u>, 462, 1960.
10. C. W. Horton and G. S. Innis, "Supplement to the Computation of Far Field Radiation Patterns from Measurements Made Near the Source," Defense Research Laboratory, University of Texas, Austin, Texas.
11. H. W. Marsh, "Near Field-Far Field Noise Relations," supplement of letter to M. Lasky, (Office of Naval Research) dated Feb. 15, 1963.
12. H. G. Ferris, "Calculation of the Far-Field Intensity Pattern of an Extended Noise Source," Hughes Aircraft Company, Report to M. A. Basin dated March 13, 1963.

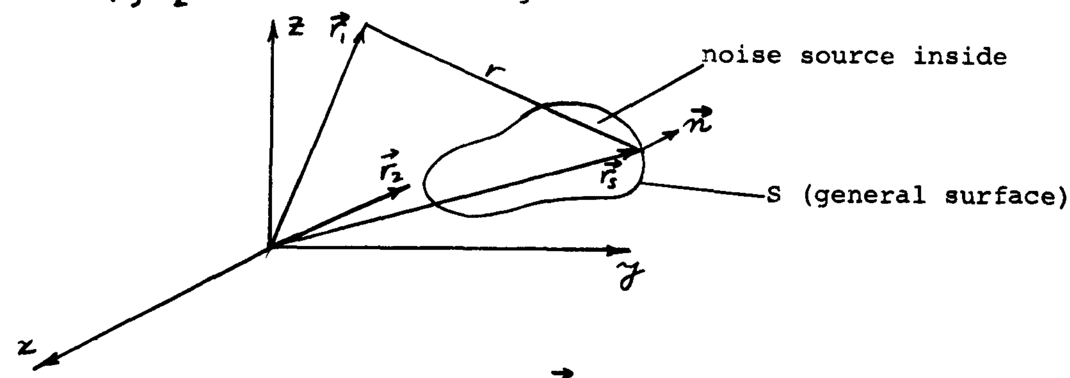
* i.e. the Green's Function for the wave equation

III. Basic equations

The cross correlation function of the noise pressure is defined as

$$\Gamma_{12}(\vec{r}_1,\vec{r}_2,\tau)=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{+T} p(\vec{r}_1,t)\,p(\vec{r}_2,t-\tau)\,dt \qquad [1]$$

where $\vec{r}_1$, $\vec{r}_2$ are shown in Fig. 1



$\vec{r}_s$ = location of any point on S

r = distance from near field point on surface to far field point at $\vec{r}_1$

$\vec{n}$ = outward normal

It has been shown[5-7] that Γ_{12} satisfies the two wave equations

$$c^2\nabla_1^2\Gamma_{12}=\frac{\partial^2\Gamma_{12}}{\partial\tau^2}\quad,\quad c^2\nabla_2^2\Gamma_{12}=\frac{\partial^2\Gamma_{12}}{\partial\tau^2} \qquad [2]$$

Parrent[8] has carried the analysis one step further and introduced the Fourier Transform of Γ_{12} i.e.

$$G_{12}(\vec{r}_1,\vec{r}_2,\nu)=\int_{-\infty}^{+\infty}\Gamma(\vec{r}_1,\vec{r}_2,\tau)\,e^{-2\pi i\nu\tau}\,d\tau \qquad [3]$$

This Fourier Transform in acoustics is actually the cross power spectrum of the sound pressure at points $\vec{r}_1$ and $\vec{r}_2$. It satisfies the scalar Helmoltz equations

$$[\nabla_\alpha^2+k^2]\,G_{12}(\vec{r}_1,\vec{r}_2,\nu)=0 \qquad \alpha=1,2 \qquad [4]$$

The problem at hand is to calculate the power spectrum of the far field pressure. Since our system is of the constant parameter linear type, we can employ the relation between the cross power spectrum and the power spectra at two points. If $G_1(\vec{r}_1,\nu)$ is the power spectrum at $\vec{r}_1$ (far field) and $G_2(\vec{r}_2,\nu)$ is the power spectrum at $\vec{r}_2$ (reference point in near field) then[13]

$$G_1(\vec{r}_1,\nu) = \frac{|G_{12}(\vec{r}_1,\vec{r}_2,\nu)|^2}{G_2(\vec{r}_2,\nu)} \qquad [5]$$

The power spectrum of the near field reference point $G_2(\vec{r}_2,\nu)$ can be obtained directly from measurements. The cross power spectrum $G_{12}(\vec{r}_1,\vec{r}_2,\nu)$ satisfies the scalar Helmholtz equations [4] so that[2,8]

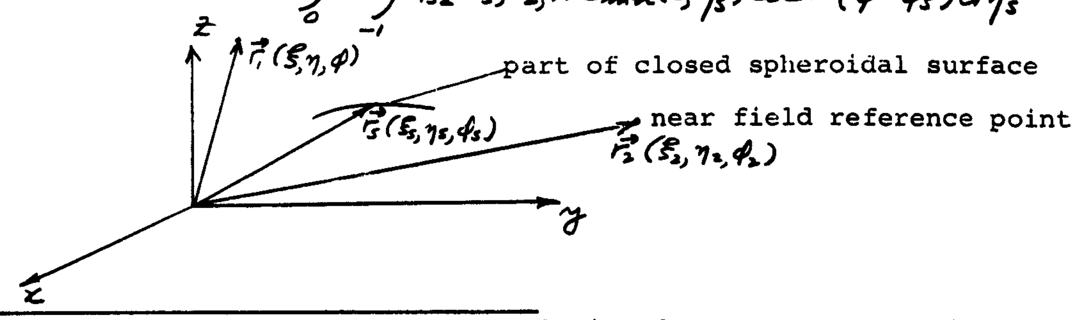
$$G_{12}(\vec{r}_1,\vec{r}_2,\nu) = -\frac{1}{4\pi}\int_S G_{S2}(\vec{r}_s,\vec{r}_2,\nu)\frac{\partial g_1}{\partial n}\,dS \qquad [6]$$

where $G_{S2}(\vec{r}_s,\vec{r}_2,\nu)$ is the cross power spectrum between a point on the surface at $\vec{r}_s$ and the near field reference point at $\vec{r}_2$. g_1 is the Green's Function which vanishes over the surface S.

For the most important practical problems which face us now, a spheroidal surface is the most logical closed surface to choose over which to make the measurements. Using the results of Horton-Innis[2] the expression for the cross power spectrum $G_{12}(\vec{r}_1,\vec{r}_2,\nu)$ can be written down immediately in terms of spheroidal wave functions.[14]

$$G_{12}(\vec{r}_1,\vec{r}_2,\nu) = \frac{1}{2\pi}\sum_{m=0}^{\infty}\sum_{n=m}^{\infty}\frac{2-\delta_{0m}}{N_{mn}}S_{mn}(c,\eta)\frac{R^{(3)}_{mn}(c,\xi)}{R^{(3)}_{mn}(c,\xi_s)} \qquad [7]$$

$$\times\int_0^{2\pi}d\phi_s\int_{-1}^{+1}G_{S2}(\vec{r}_s,\vec{r}_2,\nu)S_{mn}(c,\eta_s)\cos m(\phi-\phi_s)\,d\eta_s$$



13. J. S. Bendat, "Measurement and Analysis of Power Spectra and Cross Power Spectra for Random Phenomena," Thompson Ramo-Wooldridge, Inc., WADD TR 60-681 (Wright Air Dev. Div.) P. 96.
14. For all notation on spheroidal wave functions see C. Flammer, "Spheroidal Wave Functions," Stanford Un. Press, Stan., Cal., 1957.

Equations [5] and [7] are the basic equations which are used to compute the far field noise power spectrum. The cross power spectrum $G_{s2}(\vec{r}_s, \vec{r}_2, \nu)$ between the near field reference point and the surface points is readily obtained from measurements. For <u>single frequency</u> oscillations of frequency ν the far field pressure is obtained directly from eq. [7][2] by replacing $G_{12}(\vec{r}_1, \vec{r}_2, \nu)$ by $p(\vec{r}_1, \nu)$ (the pressure at the far field point) and $G_{s2}(\vec{r}_s, \vec{r}_2, \nu)$ by $p(\vec{r}_s, \nu)$ (the pressure on the surface). Thus all the mathematical development in spheroidal waves can be used for single frequency as well as for noise.

In principal the solution is complete, but practically speaking it cannot be used unless computer programs for obtaining spheroidal wave functions are available. Several investigators[15,16,17] have recently done work on these functions and the possibilities of having an accurate as well as rapid computer program within the near future seems very promising.

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15. S. Hanish, Naval Research Laboratory, has developed a NAREC program for computing the spheroidal wave functions. This NAREC program cannot be used directly in a near field—far field composite program because the NAREC is only available at NRL and is a comparatively slow machine.

16. T. Theilheimer, Lt. Stuckey and the group at Applied Mathematics Laboratory at the David Taylor Model Basin are developing programs for computing the eigen values of spheroidal wave functions as well as the Legendre Polynomials and Spherical Bessel Functions.

17. A. Silbiger, "Asymptotic Formulas and Computational Methods for Spheroidal Wave Functions," Report U-123-48, Cambridge Acoustical Associates, Inc., October, 1961.